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Heterotic–type IIA duality with fluxes

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Abstract: In this paper we study a possible non-perturbative dual of the heterotic string compactified on $K3 \times T^2$ in the presence of background fluxes. We show that type IIA string theory compactified on manifolds with SU(3) structure can account for a subset of the possible heterotic fluxes. This extends our previous analysis to a case of a non-perturbative duality with fluxes.

Keywords: String Duality, Flux compactifications.

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Contents

1. Introduction

Compactifications with fluxes have a received some attention recently for a number of different reasons [1]. For example, they are often necessary in the construction of string backgrounds which include non-trivial D-branes. At the same time it has been realized that they generate a potential for (some of) the scalar moduli present in all supersymmetric string compactifications. This in turn can lift part of the vacuum degeneracy of a given string background and lead to more realistic scenarios of particle phenomenology or cosmology.

A geometrical 'cousin' of flux compactifications are string backgrounds with a space-time which is of the form $M^{1,3} \times \hat{Y}$ where $\hat{Y}$ is a compact six-dimensional manifold with G-structure. Such manifolds are natural generalizations of Calabi-Yau and/or G_2 manifolds in that they also admit a globally defined spinor but this spinor is not covariantly constant. As a consequence a scalar potential is induced and, exactly as for flux backgrounds, part of the vacuum degeneracy is lifted [1].

One of the interesting questions concerning these new backgrounds is the fate of perturbative and non-perturbative dualities which hold for compactifications on Calabi-Yau manifolds. For example, it has been shown in refs. [2]–[9] that mirror symmetry between

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type IIA and type IIB string theories compactified on mirror manifolds can be maintained in the presence of background fluxes if the compactification manifold is chosen to be within a specific class of manifolds with SU(3) structure. Similar considerations have been carried out for a variety of other backgrounds including geometrical [10]–[16] and non-geometrical set-ups [17]–[28].

Mirror symmetry is a perturbative duality in that it does not act on the dilaton and holds at weak coupling of both mirror symmetric backgrounds. However, it is of obvious interest to also study non-perturbative dualities in the presence of background fluxes and/or for compactifications on manifolds with G-structure. In this case the dilaton is non-trivially involved in the duality map and hence the analysis becomes more complicated.

In this paper we study an example of a 'generalized' non-perturbative duality. We consider a subset of possible fluxes in compactifications of the heterotic string on $K3 \times T^2$ and show that a candidate dual is the type IIA string compactified on manifolds with SU(3) structure. This generalizes the non-perturbative duality between the heterotic string on $K3 \times T^2$ and type IIA strings compactified on Calabi-Yau threefolds [29]–[33]. A first step in this direction was undertaken in ref. [34] where it was shown that the dual of the Abelian gauge field strength fluxes through a certain cycle on the heterotic side corresponds to turning on (electric) RR fluxes on the type IIA side. In this paper we turn on a different set of fluxes in the heterotic string and argue that their type IIA dual corresponds to the torsion of a SU(3)-structure manifold considered previously in refs. [6, 8, 35]. We perform the analysis at the level of the $N = 2$ effective action for a whole class of such compactifications.

The paper is organized as follows. In section 2.1 we briefly recall the necessary facts about the heterotic string compactified on $K3 \times T^2$ with background fluxes following [36]. In 2.2 we slightly generalize our previous analysis in that we choose a more general solution to the Bianchi identity of the NS B-field. This in turn leads to a more general form of the resulting potential. In section 3 we propose a non-perturbative dual compactification which consists of the type IIA string compactified on a specific class manifolds with SU(3) structure. We compute the effective action and in particular the Killing vectors and the potential. In section 4 we argue that the two actions are equivalent and section 5 contains our conclusions. In order to make the paper self-contained we include two appendices with some well known facts about heterotic and type IIA compactifications with $N = 2$ supersymmetry in four dimensions. In both appendices we also display some of the more technical details which are needed in order to show the consistency with $N = 2$ gauged supergravity of the compactifications studied in the main text.

2. The Heterotic string compactified on $T^2 \times K3$

In this section we briefly review the compactification of the heterotic string on the six-dimensional manifold $T^2 \times K3$ with background fluxes following [36]. However, we do not consider the most general set of fluxes but instead focus only on fluxes for U(1) gauge fields along non-trivial two-cycles of the $K3$ manifold. The reason is that for this set of fluxes we are able to identify a type IIA dual background. In 2.1 we recall the results of [36] while in 2.2 we slightly generalize our previous analysis by allowing for a more general solution of the B-field Bianchi identity.

2.1 The effective action

The low energy limit of the ten-dimensional heterotic string is described by $N = 1$ supergravity coupled to $N = 1$ super Yang-Mills theory with gauge group SO(32) or $E_8 \times E_8$. The massless bosonic fields of the gravitational multiplet are the metric g_{MN}, the anti-symmetric tensor field B_{MN} and the dilaton φ, while the gauge fields A^a_M are members of vector multiplets. The index a runs over the adjoint representation of SO(32) or $E_8 \times E_8$ but for our purpose the specific choice of the gauge group is not relevant and therefore we discuss both cases simultaneously.

The absence of anomalies in the ten-dimensional supergravity requires that the B-field participates in a Green-Schwarz mechanism which leads to a modification of its field strength H by appropriately normalized Lorentz- and Yang-Mills Chern-Simons terms

$$H = dB + \omega_{\rm L} - \omega_{\rm YM} \ . \tag{2.1}$$

As a consequence the Bianchi identity reads

$$dH = trR \wedge R - trF \wedge F \, , \tag{2.2}$$

where R is the Riemann curvature tensor interpreted as a two form with values in the Lie algebra of the local Lorenz group while F represent the field strengths of the gauge fields. The details of the compactification (e.g. the precise light spectrum) depend on the way in which this condition is implemented. Since we are mainly concerned with the computation of the potential we do not need to specify a particular solution of (2.2) here. For our purposes it is sufficient to assume that (2.2) is satisfied, for example, by using the standard embedding. Whichever solution we choose it will generically break the gauge group to some subgroup G of SO(32), or $E_8 \times E_8$. For concreteness, we consider turning on an instanton configuration, $F_{\rm inst}$, on $K3$ with instanton number $\int_{K3} F_{\rm inst} \wedge F_{\rm inst} = 24$, so that this cancels the contribution from the second Chern class, $\int_{K3} R \wedge R = 24$, in (2.2).

The $K3$ factor in the compactification breaks half of the 16 supercharges of the ten-dimensional theory. As a consequence the four-dimensional effective theory has 8 supercharges corresponding to $N = 2$ supersymmetry. The massless spectrum contains the gravitational multiplet with metric, gravitinos and graviphoton as components. In addition there can be n_v vector multiplets each with a vector, two gauginos and a complex scalar and n_h hypermultiplets which contain four real scalars and two hyperinos.

A $K3$ manifold has 58 geometric moduli which combine with 22 axions coming from the internal B-field to form 20 hypermultiplets. The 80 scalars of these multiplets span the quaternionic coset manifold

$$\mathcal{M}_H = \frac{\mathrm{SO}(4, 20)}{\mathrm{SO}(4) \times \mathrm{SO}(20)} \ . \tag{2.3}$$

$\mathcal{M}_H$ is a submanifold of the entire quaternionic manifold which is a lot more complicated and also contains moduli arising from the gauge bundle. These moduli will play no role in the following and we will most of the time set them to an arbitrary background value.

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The massless vector fields have two different origins: first of all one has the gauge fields of the unbroken gauge group G and in addition there are the Abelian Kaluza-Klein vectors of T^2. Due to the T^2 factor in the compactification, the Yang-Mills theory always has a Coulomb branch where G is broken to its maximal Abelian subgroup. In the following we assume that we are at a generic point in the moduli space and only consider n_v Abelian vector multiplets coupled to supergravity. Their scalar superpartners are the dilaton, the axion, the moduli of T^2 as well as the gauge fields on the internal torus which are scalars from a four-dimensional point of view. Altogether these fields span the coset space

$$\mathcal{M}_V = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \otimes \frac{\mathrm{SO}(2, n_v - 1)}{\mathrm{SO}(2) \times \mathrm{SO}(n_v - 1)} \ . \tag{2.4}$$

Let us now discuss the fluxes which we consider in this paper. They arise from the gauge fields on $K3$ and therefore must be disentangled from the instanton contribution.[1] As explained before, the precise instanton configuration F_{inst} which we choose in order to satisfy (2.2), generically breaks the gauge group to some subgroup G of SO(32) or $E_8 \times E_8$. By going to the Coulomb branch, the gauge group is further broken to the maximal Abelian subgroup of G. From this point of view the non-Abelian nature of the ten-dimensional gauge group is only relevant for solving the constraint (2.2) and once it is solved we can discard these gauge fields together with all other fields which become massive in this process. Now the fluxes we turn on are precisely for the 'left-over' Abelian gauge fields on $K3$ which also include the Kaluza-Klein vectors of T^2. More precisely, we assume that the following integrals are non-trivial

$$\int_{\gamma^\alpha} F^I_{\text{flux}} = m^{\alpha I} \ , \qquad \alpha = 1, \ldots, 22 \ , \tag{2.5}$$

where γ^α denotes the 22 non-trivial two-cycles of $K3$ and the index $I = 0, \ldots, n_v$ label the Abelian vector fields on the Coulomb branch.[2] Instead of defining the flux parameters $m^{I\alpha}$ via the integral (2.5) we can equally well expand F^I_{flux} in terms of a (real) basis of harmonic two-forms ω^α on $K3$ which are dual to the cycles γ^α. This amounts to $F^I_{\text{flux}} = m^{I\alpha}\omega_\alpha$.

The only thing we have to take care of at this point is that the fluxes can also contribute to the right hand side of the Bianchi identity (2.2). This contribution — when integrated over $K3$ — is given by [37]

$$\delta = \int_{K3} F^I_{\text{flux}} \wedge F^J_{\text{flux}} \eta_{IJ} = m^{\alpha I} m^{\beta J} \rho_{\alpha\beta} \eta_{IJ} \ , \tag{2.6}$$

where $\rho_{\alpha\beta}$ denotes the $K3$ intersection matrix, $\rho_{\alpha\beta} = \int \omega_\alpha \wedge \omega_\beta$, which has signature $(3, 19)$, while η_{IJ} is the invariant tensor on the $\mathrm{SO}(2, n_v - 1)$ factor of the moduli space defined in (2.4) which has signature $(2, n_v - 1)$. Within the set-up we have presented so far, δ has to vanish for consistency. The reason being that at this point we have already assumed that F_{inst} saturates the constraint (2.2) and the fluxes should not spoil this solution. In the next section we relax this constraint but for now we impose $\delta = 0$.

[1] We thank André Lukas for drawing our attention on this point.

[2] $I = 0$ counts the graviphoton which is an Abelian vector in the theory but resides in the gravitational rather than in a vector multiplet.

Without background fluxes the low energy effective action for this compactification corresponds to an ungauged $N = 2$ supergravity as it has been computed in refs. [38, 39]. Turning on background fluxes gauges some of the isometries of the moduli space and generates a potential. One assumes that the fluxes are turned on adiabatically so that the light spectrum does not change and that both the string scale and the Kaluza-Klein scale are well above the scale set by the fluxes. This ensures the consistency of the compactification.

The low energy effective action for this compactification was derived in [36] and shown to have the general form of an $N = 2$ gauged supergravity. The bosonic terms are found to be

$$S = \int \Big[\frac{1}{2}R*\mathbf{1} - g_{i\bar{\jmath}}dx^i\wedge *d\bar{x}^{\bar{\jmath}} - h_{uv}Dq^u\wedge *Dq^v + \frac{1}{4}\,\mathrm{Im}\mathcal{N}_{IJ}F^I\wedge *F^J + \frac{1}{4}\,Re\mathcal{N}_{IJ}F^I\wedge F^J - V\Big]\,, \tag{2.7}$$

where the $x^i, i = 1, \ldots, n_v$ denote the complex scalars in the vector multiplets, whose precise definition is given in appendix A, while q^u are the $4n_h$ real scalars of the hypermultiplets. The $F^I = dA^I$ are the Abelian gauge field strength and $\mathcal{N}_{IJ}$ is the $N = 2$ gauge coupling matrix given explicitly in (A.9). h_{uv} is the quaternionic metric on the hypermultiplet moduli space which in general is unknown. For the subset of the 80 $K3$ moduli it is the metric on the coset $\mathrm{SO}(4, 20)/\,\mathrm{SO}(4) \times \mathrm{SO}(20)$. Finally, $g_{i\bar{\jmath}} = \partial_{x^i}\partial_{\bar{x}^{\bar{\jmath}}}K$ is the (special) Kähler metric of the vector multiplet moduli space with K given in (A.2).

If no fluxes are turned on, the covariant derivatives Dq^u coincide with partial derivatives and the potential V in (2.7) vanishes. Turning on the fluxes (2.5) gauges the Peccei-Quinn isometries associated with the scalars coming from the B field on $K3$. This is easy to see from the fact that gauge invariance of H requires the B-field to transform non-trivially in order to obey (2.1). More precisely, for an Abelian gauge transformation, $\delta A^I = d\lambda^I$ one needs the compensating transformation $\delta B = \lambda^I F^J \eta_{IJ}$. Expanding the B-field along $K3$ yields the 22 axionic scalars b^α from[3]

$$\hat{B} = B + b^\alpha \omega_\alpha\,, \tag{2.8}$$

where B is an antisymmetric tensor in the four-dimensional space-time. For the case of non-trivial background flux (2.5), we see immediately that the b^α transform as $\delta b^\alpha = m^{I\alpha}\lambda^J\eta_{IJ}$ and therefore the effective action has to include the covariant derivative

$$Db^\alpha = db^\alpha - m_I^\alpha A^I\,, \tag{2.9}$$

where we have defined

$$m_I^\alpha = \eta_{IJ}m^{\alpha J}\,. \tag{2.10}$$

Apart from the covariant derivatives also a potential V is generated by the fluxes. It arises from the kinetic term for the gauge fields and therefore has the form [36]

$$V_{\text{flux}} = -\frac{1}{2}h_{\alpha\beta}\, m_I^\alpha m_J^\beta\, \left(\mathrm{Im}\mathcal{N}^{-1}\right)^{IJ}\,, \tag{2.11}$$

[3] We have introduced a hat, $\hat{}$, in order to distinguish the ten-dimensional B-field from the four dimensional one.

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where $h_{\alpha\beta}$ is the restriction of the quaternionic metric h_{uv} to the space spanned by the charged scalars. As we just argued these are precisely the axionic scalars b^α and thus their σ-model metric can be derived from a direct reduction of the kinetic term of the B-fields. This yields

$$h_{\alpha\beta} = \frac{1}{4v} \int_{K3} \omega_\alpha \wedge *\omega_\beta \,, \tag{2.12}$$

where v denotes the volume of $K3$.

To summarize, the fluxes gauge the $N = 2$ supergravity in that they induce the covariant derivatives (2.9) and the potential (2.11). The consistency with $N = 2$ gauged supergravity was shown in [36]. However, in that proof it was essential that δ defined in (2.6) vanishes. It is this last constraint which we now want to relax.

2.2 Generalized fluxes

So far we have reviewed the results obtained in [36] for heterotic compactifications with fluxes. In particular, we chose the fluxes to obey $\delta = 0$ where δ was defined in (2.6). In this section we slightly generalize the setup in that we also consider fluxes for which $\delta \neq 0$ holds. The important point to note is that requiring δ in (2.6) to vanish is quite an arbitrary choice of having the Bianchi identity (2.2) satisfied. The full integrated Bianchi identity in fact reads

$$0 = \int_{K3} \left[tr(F_{\text{inst}} \wedge F_{\text{inst}}) - tr(R \wedge R) \right] + \delta \,. \tag{2.13}$$

Demanding that both the integral as well as δ vanish separately is only a special solution. Generically δ can be arbitrary as long as we choose F_{inst} in such a way that the above equation is satisfied. Let us take for now this point of view and discuss fluxes with arbitrary δ and assume that the integral in (2.13) is appropriately chosen such that this condition is satisfied.

We see immediately that the covariant derivatives given in (2.9) do not depend on the value of δ and therefore remain unchanged. However, the potential (2.11) does change. Additional terms can arise from higher derivative (α') corrections including (R^2)-terms in the ten-dimensional action. For a background which is Ricci flat — as it is our case — the only non-vanishing terms are contractions of the Riemann curvature tensor with itself. This combines with the kinetic term for the gauge fields into

$$S_{FR} = -\frac{1}{2} \int e^{-\varphi} \Big[trF \wedge *F - trR \wedge *R \Big] \,, \tag{2.14}$$

where we have set $\alpha' = 1$. Let us now compute the contribution of the above term to the potential in an instantonic background which satisfies (2.13) for some arbitrary δ. For this we note that in order to preserve supersymmetry F_{inst} has to be of complex type $(1,1)$ and primitive [40]. For a general $(1,1)$ form on a two-dimensional complex manifold we can write

$$(*F)_{a\bar{b}} = \epsilon_{a\bar{b}}{}^{c\bar{d}} F_{c\bar{d}} = \epsilon_{ad} \epsilon_{\bar{b}}{}^{c} F_c{}^d = \frac{1}{2}(g_{a\bar{b}} \delta^c_d - g_{d\bar{b}} \delta^c_a) F_c{}^d = \frac{1}{2} g_{a\bar{b}} F^c_c - \frac{1}{2} F_{a\bar{b}} \,, \tag{2.15}$$

where $g_{a\bar{b}}$ is the metric on $K3$ written in complex coordinates and we have used the decomposition of the four-dimensional ϵ-symbol under complex indices $\epsilon_{ab}{}^{cd} = \epsilon_{ab}\epsilon^{cd} = \delta^{cd}_{ab}$. The primitivity condition for $F_{\rm inst}$ further implies that $(F_{\rm inst})_a{}^a = 0$ which inserted in (2.15) implies

$$*F_{\rm inst} = -\frac{1}{2}F_{\rm inst} \ . \tag{2.16}$$

The same is true for the curvature two-form. $K3$ is a Kähler manifold and thus, R is of type $(1,1)$. In addition, $K3$ is Ricci flat, which implies the primitivity condition. Therefore, the contribution of the integral (2.14) to the potential, in an instantonic background satisfying (2.13), is

$$\begin{aligned} V_{\rm inst} &= \frac{1}{2}\int_{K3} e^{-\varphi}\Big[trF_{\rm inst}\wedge *F_{\rm inst} - trR\wedge *R\Big] \\ &= -\frac{1}{4}\int_{K3} e^{-\varphi}\Big[trF_{\rm inst}\wedge F_{\rm inst} - trR\wedge R\Big] = \frac{1}{4}e^{-\varphi}\delta \ , \end{aligned} \tag{2.17}$$

where we have assumed that the dilaton is constant on $K3$. We see that for a setup where the fluxes do not contribute to the Bianchi identity, i.e. $\delta = 0$, the instanton contribution to the potential precisely cancels the contribution from the term R^2 due to the curvature of the internal manifold. On the other hand, if we turn on fluxes such that $\delta \neq 0$, the instantons are no longer balanced against the internal curvature and therefore the above term has to be taken into account in the potential. Thus, the complete potential, including also the flux contribution (2.11), reads

$$V = V_{\rm flux} + V_{\rm inst} = -\frac{1}{2}\,h_{\alpha\beta}\,m^\alpha_I m^\beta_J\,({\rm Im}\mathcal{N}^{-1})^{IJ} + \frac{e^\phi}{4v}\,m^{\alpha I}m^{\beta J}\rho_{\alpha\beta}\,\eta_{IJ} \ , \tag{2.18}$$

where we have multiplied (2.17) by a factor $e^{2\phi}$ which corresponds to a transformation into the Einstein frame and we have also inserted the definition (2.6) of δ. In the derivation above we have used the fact that the ten-dimensional dilaton $e^{-\varphi}$ is multiplied by the volume V_6 of the internal manifold in order to obtain the properly normalized four-dimensional dilaton $e^{-\phi} = e^{-\varphi}V_6$. We further assumed that the integration over the two-torus can be trivially performed yielding a volume factor of T^2 which enters correctly in the definition of the four-dimensional dilaton. On the other hand, the topological $K3$ integral (2.13) does not yield a volume factor, v, of $K3$, which therefore appears explicitly in (2.18).

The next task is to show that the potential V of (2.18) together with the covariant derivatives (2.9) is consistent with $N = 2$ gauged supergravity. This check is presented in appendix A.2. Instead we now turn to our proposal for a dual type IIA background.

3. Type IIA compactified on manifolds with SU(3) structure

The goal of this section is to identify a dual type IIA background. Without fluxes heterotic compactifications on $K3 \times T^2$ are non-perturbatively dual to type IIA compactified on Calabi-Yau threefolds. This duality is non-perturbative in the sense that the dilatons in both backgrounds are mapped to geometrical moduli and thus are not constrained to be at weak coupling.[4]

A first obvious attempt is to turn on fluxes also on the type IIA side. Indeed in ref. [34] it was shown that RR fluxes on the type IIA side which charge the axion under all vector multiplets, correspond to gauge field fluxes on the heterotic side through the $\mathbf{P}^1$ base of an appropriately fibered $K3$. However, the fluxes through the other 21 two-cycles in $K3$ do not have obvious duals in the type IIA picture.

Here we are going to propose that the dual of the heterotic fluxes can arise by modifying the compactification manifold. A similar approach has already been pursued in the context of mirror symmetry. In ref. [2] it was shown that a certain class of manifolds with SU(3) structure — termed half-flat manifolds — are possible mirror duals to Calabi-Yau compactifications of type IIB with NS three-form flux. These manifolds are characterized by the existence of a globally defined and nowhere vanishing spinor η which reduces the structure group from SO(6) to SU(3). However, unlike in the Calabi-Yau case, this spinor is not covariantly constant with respect to the Levi-Civita connection, but only with respect to a connection with torsion. Or in other words the (intrinsic) torsion measures the deviation of this spinor from being covariantly constant.

The existence of the spinor implies the existence of a $(1,1)$-form, J, and a $(3,0)$ form, Ω, which are built from appropriate spinor bilinears. As a consequence of η being not covariantly constant, both J and Ω are not closed. Instead they obey

$$\begin{aligned} (dJ)_{mnp} &= \; -6T_{[mn}{}^{q}J_{p]q} \; , \\ (d\Omega)_{mnpq} &= 12T_{[mn}{}^{r}\Omega_{pq]r} \; , \end{aligned} \tag{3.1}$$

where T denotes the intrinsic torsion. In this language Calabi-Yau manifolds are manifolds with SU(3) structure for which the torsion vanishes and hence η is covariantly constant with respect to the Levi-Civita connection and both J and Ω are closed.

The existence of the spinor also ensures that the low energy effective action has $N=2$ supersymmetry. The presence of torsion gauges this supergravity and induces a scalar potential. Hence such compactifications are natural candidates for duals of flux compactifications which, as we saw in the previous section, have exactly the same effect.

Type II compactifications on half-flat manifolds were studied in refs. [2, 3], while ref. [8] considered a more general class of SU(3) structure compactifications. It is within this generalized class of manifolds that we will locate the duals of heterotic flux compactifications.

Since compactifications on manifolds with SU(3) structure have already been spelled out in some detail in refs. [2, 3, 8, 41] we will be brief in the following and only concentrate on the important points.

The next step is to derive the low energy effective action of type IIA supergravity compactified on manifolds with SU(3) structure $\hat{Y}$ as proposed in [6, 8, 35]. We start from (massless) type IIA supergravity in ten dimensions whose bosonic degrees of freedom consist of the graviton, $\hat{g}_{MN}$, an antisymmetric tensor field, $\hat{B}_2$, and the dilaton, $\hat{\phi}$, in the NS-NS sector and a one-form, $\hat{C}_1$, and a three-form, $\hat{C}_3$, in the RR sector. The ten-dimensional

[4] We briefly review this duality in section 4.

action is given by

$$S = \int e^{-2\hat{\phi}}\big(\frac{1}{2}\hat{R}*\mathbf{1} + 2d\hat{\phi}\wedge *d\hat{\phi} - \frac{1}{4}\hat{H}_3\wedge *\hat{H}_3\big) - \frac{1}{2}\left(\hat{F}_2\wedge *\hat{F}_2 + \hat{F}_4\wedge *\hat{F}_4\right)$$
$$-\frac{1}{2}\Big[\hat{B}_2\wedge d\hat{C}_3\wedge d\hat{C}_3 - (\hat{B}_2)^2\wedge d\hat{C}_3\wedge d\hat{C}_1 + \frac{1}{3}(\hat{B}_2)^3\wedge d\hat{C}_1\wedge d\hat{C}_1\Big] , \qquad (3.2)$$

where

$$\hat{F}_4 = d\hat{C}_3 - d\hat{C}_1\wedge\hat{B}_2 , \qquad \hat{F}_2 = d\hat{C}_1 , \qquad \hat{H}_3 = d\hat{B}_2 . \qquad (3.3)$$

The action is invariant under the following three independent Abelian gauge transformations

$$\begin{aligned} \delta\hat{C}_1 &= d\hat{\theta} , \qquad \delta\hat{C}_3 = d\hat{\Lambda}_2 , \\ \delta\hat{B}_2 &= d\hat{\Theta}_1 , \quad \delta\hat{C}_3 = \hat{C}_1\wedge d\hat{\Theta}_1 . \end{aligned} \qquad (3.4)$$

In order to perform the compactification we follow the strategy outlined in ref. [8]. The first step is to decompose all ten-dimensional fields under $SO(1,3)\times SU(3) \subset SO(1,9)$. This merely amounts to a rewriting of the ten-dimensional fields in a background with a smaller Lorentz group. If one identifies the $SO(1,3)$ factor with the Lorentz group of a four-dimensional space-time the resulting effective theory has $N = 8$ supersymmetry. This is most easily seen from the decomposition of the **16** spinor representation of $SO(1,9)$ under $SO(1,3)\times SU(3)$ which yields

$$\mathbf{16} \to (\mathbf{2},\mathbf{3}) \oplus (\mathbf{2},\mathbf{1}) \oplus (\bar{\mathbf{2}},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{2}},\bar{\mathbf{1}}) . \qquad (3.5)$$

Here the **2** and $\bar{\mathbf{2}}$ denote complex conjugate Weyl spinors of $SO(1,3)$ and thus we see that four supersymmetries result from the **16**-dimensional spinor representation. Since type IIA has two gravitinos in the **16**, the effective theory has $N = 8$ supersymmetry. One way to reduce the supersymmetry is to project out all $SU(3)$ triplets **3** and only keep the singlets **1**. This singlet is precisely the invariant spinor η mentioned previously. As can be seen from (3.5) such a truncation has only one gravitino from each of the two **16** or in other words $N = 2$ supersymmetry from a four-dimensional point of view. The truncation has to be implemented on the entire spectrum and, as shown in ref. [8], the resulting spectrum can be arranged in $N = 2$ multiplets.

The next step is to Kaluza-Klein expand the ten-dimensional fields in terms of a set of two-forms ω_i (with dual four-forms $\tilde{\omega}^i$) and a set of three-forms (α_A, β^B) on $\hat{Y}$.[5] These forms are not necessarily harmonic but $(1, \omega_i, \tilde{\omega}^i, \epsilon_g)$ form a non-degenerate symplectic basis on the space of all even forms (ϵ_g denotes the volume from) and (α_A, β^B) form a non-degenerate symplectic basis on the space of all three-forms. In other words they obey (B.1) and (B.2) exactly as their Calabi-Yau 'cousins'. The expansion of the ten-dimensional fields in this basis also resembles the Calabi-Yau situation reviewed in appendix B.1 and reads

$$\begin{aligned} \hat{B}_2 &= B_2 + b^i\omega_i , \\ \hat{C}_1 &= A^0 , \\ \hat{C}_3 &= C_3 + A^i\wedge\omega_i + \xi^A\alpha_A - \tilde{\xi}_A\beta^A . \end{aligned} \qquad (3.6)$$

[5] No fields arise from one-forms or five-forms since they are SU(3) triplets and projected out.

Here b^i, ξ^A and $\tilde{\xi}_A$ are scalar fields, B_2 is a two-form in four dimensions which is dual to an axion a, A^0 and A^i are vector fields and C_3 is a three-form in four dimensions which is not dynamical and dual to a constant.[6] Similarly, we expand J and Ω as

$$J = v^i \omega_i \,, \qquad \Omega = Z^A \alpha_A - \mathcal{G}_A \beta^A \,, \tag{3.7}$$

where v^i represent the analog of the Calabi-Yau Kähler moduli and $z^a = Z^A/Z^0$ are the analog of the complex structure moduli. In [8] it was further shown that $\mathcal{G}_A$ is the derivative of a prepotential $\mathcal{G}(Z)$ with respect to the projective coordinates Z^A.

Altogether these fields combine into $h^{(1,1)}$ vector multiplets, consisting of the bosonic components $(A^i, x^i = b^i + iv^i)$ and $h^{(2,1)}$ hypermultiplets featuring the scalars $(z^a, \xi^a, \tilde{\xi}_a)$.[7] In addition there is a tensor multiplet with components $(B_2, \phi, \xi^0, \tilde{\xi}_0)$ and finally A^0 is the graviphoton which sits in the $N = 2$ gravitational multiplet.

The compactification now proceeds in analogy with Calabi-Yau compactifications which we recall in appendix B.1 with the difference that the forms in which we expand the fields are no longer closed. The generic case has been discussed in ref. [8] but here we are only interested in the subclass of SU(3) manifolds which are dual to the heterotic compactifications. Thus we consider manifolds which obey the following differential relations

$$\begin{aligned} d\omega_i &= -q_i^A \alpha_A \,, \\ d\alpha_A &= 0 \,, \quad d\beta^A = q_i^A \tilde{\omega}^i \,, \\ d\tilde{\omega}^i &= 0 \,, \end{aligned} \tag{3.8}$$

where q_i^A is a constant $n_v \times n_h = h^{(1,1)} \times (h^{(1,2)} + 1)$ matrix. Using (3.7) this amounts to $dJ = -v^i q_i^A \alpha_A$ and $d\Omega = \mathcal{G}_A q_i^A \tilde{\omega}^i$. The motivation for the choice (3.8) is that the q_i^A carry one index A which labels the hypermultiplets, and one index i which labels the vector multiplets. This is precisely the behavior we found in the heterotic flux compactification of the previous section.

The derivation of the effective action proceeds analogously to the Calabi-Yau case and yields a gauged $N = 2$ supergravity of the form (2.7). The kinetic terms have exactly the same form as for Calabi-Yau compactifications which we recall in appendix B.1. In particular, g_{ij} is the metric on the space of vector multiplet scalars $x^i = b^i + iv^i$ which has the form (B.5), while $\mathcal{N}_{IJ}$ are the gauge couplings which have the form (B.7) and can be obtained from the prepotential (B.6). Moreover, the quaternionic metric, h_{uv}, has precisely the form (B.9) with the matrix $\mathcal{M}$ defined as in (B.8).

The effect of the torsion, manifested through the derivatives (3.8), is to turn some of the ordinary derivatives into covariant derivatives and induce a potential. The covariant derivatives can be read off easily from the gauged isometries. Consider the gauge transformation of the three-form potential $\hat{C}_3$ in ten dimensions

$$\delta \hat{C}_3 = d\Lambda_2 \,, \tag{3.9}$$

[6] Later on it will be essential to properly perform this dualization as it generates terms which contribute to the potential.

[7] We are abusing the notation here in that we denote the dimension of the set of two-forms by $h^{(1,1)}$ and the dimension of the set of three-form by $2 + 2h^{(2,1)}$.

and expand the two-form gauge parameter in the two-forms ω_i

$$\Lambda_2 = \lambda^i \omega_i \ . \tag{3.10}$$

Using the differential relations (3.8) and inserting the gauge transformation in the expansion (3.6) we obtain the following transformation properties

$$\delta\xi^A = -\ \lambda^i q_i^A \ , \qquad \delta A^i = \quad d\lambda^i \ . \tag{3.11}$$

The corresponding covariant derivative which is invariant under this transformation reads

$$D\xi^A = d\xi^A + q_i^A A^i \ . \tag{3.12}$$

Of course, the same conclusion can be reached by computing the gauge invariant field strength $\hat{F}_4$ using the expansion (3.6)

$$\hat{F}_4 = dC_3 - dA^0 \wedge B_2 + (dA^i - dA^0 b^i)\wedge\omega_i + (d\xi^A + q_i^A A^i)\wedge\alpha_A - d\tilde{\xi}_A \wedge \beta^A - \tilde{\xi}_A q_i^A \tilde{\omega}^i \ . \tag{3.13}$$

Having found the charged fields, let us turn to the quaternionic metric (B.9). Notice that in these kinetic terms, the charged fields ξ^A appear also without a derivative and in order to render this term gauge invariant we need to introduce a further term which contains the gauge fields A^i. However, this can be avoided if we perform a field redefinition $a \to a - \xi^A \tilde{\xi}_A$ under which (B.9) becomes

$$\begin{aligned} h_{uv} Dq^u \wedge *Dq^v = \ & \frac{1}{4}(d\phi)^2 + g_{a\bar{b}} dz^a \wedge * d\bar{z}^b + \frac{e^{4\phi}}{4}\left[da - 2\tilde{\xi}_A D\xi^A\right]^2 \\ & - \frac{e^{2\phi}}{2}\left(\mathrm{Im}\mathcal{M}^{-1}\right)^{AB}\left[D\tilde{\xi}_A - \mathcal{M}_{AC} D\xi^C\right] \wedge * \left[D\tilde{\xi}_B - \bar{\mathcal{M}}_{BD} D\xi^D\right] , \end{aligned} \tag{3.14}$$

where the covariant derivatives for the fields ξ^A are given in (3.12).

By comparing (2.9) with (3.12) we see that the type IIA graviphoton does not appear in (3.12) while it does in (2.9). Or in other words the type IIA side lacks $(h^{(2,1)} + 1)$ parameters which are duals to m_0^α. From ref. [8] we know that the NS-NS fluxes combine with the torsion parameters (3.8) as the zeroth component q_0^A, while from ref. [42] we learn that the NS-NS fluxes result in gauge charges with respect to the graviphoton. These observations suggest that we also need to turn on half of the NS-NS fluxes in order to recover all the flux parameters of the heterotic compactification. Thus we choose

$$H_{\text{flux}} = -q_0^A \alpha_A \ . \tag{3.15}$$

Note that the Bianchi identity, $dH = 0$, does not impose any additional constraint but is automatically satisfied due to (3.8).

The correct treatment of such a flux requires that we perform a field redefinition in the ten-dimensional type IIA action (3.2) so that only $H = dB$ appears in the action and not the bare two-form potential B. However in this basis it is more difficult to read off the correct vector degrees of freedom in four dimensions and the calculation becomes more involved. Therefore in order not to over complicate the calculation we shall set the fluxes (3.15) to zero at the beginning and switch them back on at the very end using the following observation. Due to the differential relations (3.8) a shift in the vacuum expectation value for the scalars b^i in (3.6) of the form $b^i \to b^i + \rho^i$ will generate an H-flux of the form $H_{\text{ind}} = -\rho^i q_i^A \alpha_A$, which is precisely the flux (3.15) if we identify[8]

$$q_0^A = \rho^i q_i^A \ . \tag{3.16}$$

The important thing to note here is that the b^i-shift is not a symmetry of the theory since, due to (3.8), H shifts as specified above. If the ω_i were closed, the b^i-shift would be a symmetry of the action and the b^i would be true axions of the low energy theory. As ω_i are not closed the b^i-shift has to be accompanied by changes in other fields. It can be easily seen from the gauge transformation (3.4) that we need to perform an additional transformation on the ten-dimensional field $\hat{C}_3$ which is of the form $\delta\hat{C}_3 = \rho^i \omega_i \wedge A^0 = q_0^A A^0 \wedge \omega_i$. Since ω_i obeys (3.8), the field strength $\hat{F}_4$ changes accordingly by $\delta\hat{F}_4 = q_0^A A^0 \wedge \alpha_A$. Compared to the expansion (3.13) one immediately sees that this amounts to a change in the covariant derivative (3.12) for the fields ξ^A, which can now be written

$$D\xi^A = d\xi^A + q_I^A A^I \ . \tag{3.17}$$

Note that the index I now runs over all the vector fields in the theory, including the graviphoton, as argued before based on the results in [42].

Let us turn to the computation of the potential which is generated by the torsion (3.8) in the absence of H-fluxes. There will be several contributions to the potential which we shall analyze separately in the following. First of all one notices that the relations (3.8) effectively induce fluxes for the field strengths $\hat{F}_4$ and $\hat{H}_3$. From the expansion (3.6) we find

$$(\hat{F}_4)_{\text{ind}} = -\tilde{\xi}_A q_i^A \tilde{\omega}^i \ , \qquad (\hat{H}_3)_{\text{ind}} = -b^i q_i^A \alpha_A \ . \tag{3.18}$$

Clearly, these induced fluxes will generate potential terms when inserted in the corresponding kinetic terms of (3.2). Additional contributions arise from dualizing the three-form C_3 and from the fact that manifolds with SU(3) structure described by the differential relations (3.8) are in general not Ricci flat. Let is analyze these contributions one by one.

(i) Internal fluxes

Inserting (3.18) into the kinetic terms for $\hat{C}_3$ and $\hat{B}_2$ of the type IIA action (3.2) and after performing the integration over the internal manifold we find

$$V_F = \frac{e^{4\phi}}{8\mathcal{K}} \, \tilde{\xi}_A \tilde{\xi}_B q_i^A q_j^B g^{ij} \ , \tag{3.19}$$

and

$$V_H = -\frac{e^{2\phi}}{4\mathcal{K}} \, \mathcal{M}_{AC} \left(\text{Im}\mathcal{M}^{-1}\right)^{CD} \bar{\mathcal{M}}_{DB} \, q_i^A q_j^B b^i b^j \ , \tag{3.20}$$

[8]Note that if the rank of the matrix q_i^A is not at least $h^{(2,1)}+1$ we can not in general absorb all the fluxes q_0^A into the shifts ρ^i. However, in deriving the effect of these shifts we do not use arguments related to the rank of q_i^A and therefore we can consider that $rank(q_i^A) \geq h^{(2,1)}+1$ and the result will also apply to the case $rank(q_i^A) \leq h^{(2,1)}$.

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where the matrix $\mathcal{M}$ is defined in complete analogy with Calabi-Yau threefolds by the integrals given in (B.8).

(ii) Dualization of C_3 in four dimensions

After collecting all terms containing C_3 one can follow the standard procedures for dualizing a three-form in four dimensions (see e.g. [42]). Due to the non-trivial couplings of C_3, the dualization yields several terms one of which contributes to the potential.[9] We find

$$V_{C_3} = \frac{e^{4\phi}}{2\mathcal{K}} (q_i^A b^i \tilde{\xi}_A)^2 \, . \tag{3.21}$$

(iii) Internal scalar curvature

The Ricci scalar for the manifolds with SU(3) structure which we consider here was computed in [43]. After integrating over the internal manifold one obtains

$$V_R = -\frac{e^{2\phi}}{4\mathcal{K}} \mathcal{M}_{AC} \left(\mathrm{Im}\mathcal{M}^{-1}\right)^{CD} \bar{\mathcal{M}}_{DB}\, q_i^A q_j^B v^i v^j + \frac{e^{2\phi}}{2\mathcal{K}} e^{K(z)}\, \mathcal{G}_A \bar{\mathcal{G}}_B q_i^A q_j^B (g^{ij} - 4v^i v^j) \, , \tag{3.22}$$

where $\mathcal{G}_A$ are defined in (3.7).

Putting the above contributions together we obtain the final form of the potential

$$\begin{aligned} V &= V_F + V_H + V_{C_3} + V_R \\ &= -\frac{e^{2\phi}}{4\mathcal{K}} \mathcal{M}_{AC} \left(\mathrm{Im}\mathcal{M}^{-1}\right)^{CD} \bar{\mathcal{M}}_{DB}\, q_i^A q_j^B (v^i v^j + b^i b^j) \\ &\quad + \frac{e^{4\phi}}{2} \tilde{\xi}_A \tilde{\xi}_B q_i^A q_j^B \left(\frac{g^{ij}}{4\mathcal{K}} + \frac{b^i b^j}{\mathcal{K}}\right) + \frac{e^{2\phi}}{2\mathcal{K}} \mathcal{G}_A \bar{\mathcal{G}}_B q_i^A q_j^B (g^{ij} - 4v^i v^j) \, . \end{aligned} \tag{3.23}$$

In order to make the comparison with the heterotic side and also the relation to the $N=2$ supergravity form of the potential more transparent, let us regroup the terms and write it as

$$\begin{aligned} V = &\left[-\frac{e^{2\phi}}{4}\left(\mathcal{M}\mathrm{Im}\mathcal{M}^{-1}\bar{\mathcal{M}}\right)_{AB} + \frac{e^{4\phi}}{2}\tilde{\xi}_A\tilde{\xi}_B\right]\left(\frac{g^{ij}}{4\mathcal{K}} + \frac{b^i b^j}{\mathcal{K}}\right) q_i^A q_j^B \\ &+ \left[\frac{e^{2\phi}}{16\mathcal{K}}\left(\mathcal{M}\mathrm{Im}\mathcal{M}^{-1}\bar{\mathcal{M}}\right)_{AB} + \frac{e^{2\phi}}{2\mathcal{K}} e^{K(z)} \mathcal{G}_A \bar{\mathcal{G}}_B\right] \left(g^{ij} - 4v^i v^j\right) q_i^A q_j^B \, . \end{aligned} \tag{3.24}$$

Recall that the potential above was computed in the absence of H-fluxes. However, their inclusion at the level of the potential is straightforward. We have argued above that H-fluxes characterized by the parameters q_0^A in (3.16) can be turned on by incorporating the shift $b^i \to b^i + \rho^i$ with ρ^i obeying (3.16). Clearly, under this transformation only the first term in the potential (3.24) changes and we can write the potential in its final form

$$\begin{aligned} V = &-\left[-\frac{e^{2\phi}}{4}\left(\mathcal{M}\mathrm{Im}\mathcal{M}^{-1}\bar{\mathcal{M}}\right)_{AB} + \frac{e^{4\phi}}{2}\tilde{\xi}_A\tilde{\xi}_B\right]\left(\mathrm{Im}\mathcal{N}^{-1}\right)^{IJ} q_I^A q_J^B \\ &- e^{2\phi}\left[\frac{1}{2}\left(\mathcal{M}\mathrm{Im}\mathcal{M}^{-1}\bar{\mathcal{M}}\right)_{AB} + 4e^{K(z)}\mathcal{G}_A\bar{\mathcal{G}}_B\right]\left(\frac{1}{2}(\mathrm{Im}\mathcal{N}^{-1})^{IJ} + 4X^I\bar{X}^J\right) q_I^A q_J^B \, , \end{aligned} \tag{3.25}$$

[9]The dualization also introduces an additional flux parameter –the constant dual to C_3 — but for our purposes here we can set it to zero from the beginning.

where $\mathrm{Im}\mathcal{N}^{-1}$ is the inverse of the matrix given in (B.7) which has the form

$$(\mathrm{Im}\mathcal{N})^{-1} = -\frac{1}{\mathcal{K}} \begin{pmatrix} 1 & b^i \\ b^j & \frac{g^{ij}}{4} + b^i b^j \end{pmatrix} . \tag{3.26}$$

Note that we have also rewritten the second line in (3.24) in terms of $\mathrm{Im}\mathcal{N}^{-1}$ in order to make the symplectic structure of the potential manifest. In this form the comparison with gauged supergravity and with the heterotic potential will become more transparent.

The final task is to establish the consistency of the potential with the general form of V in $N = 2$ gauged supergravity as we did on the heterotic side in appendix A.2. The analogous check for the type IIA potential (3.25) is done in appendix B.2. In the next section we instead turn to the comparison between the heterotic and the type IIA action.

4. Duality of heterotic and type II compactification

In this section we want to argue that the conjectured non-perturbative duality between the heterotic string compactified on $K3 \times T^2$ and the type IIA string compactified on $K3$-fibred Calabi-Yau threefolds, [29 − 33], can be generalized to a duality between the heterotic string compactified on $K3 \times T^2$ in the presence of a specific set of fluxes and the type IIA string compactified on a particular subclass of manifolds with SU(3) structure. We discuss this at the level of the bosonic effective actions which we derived in the previous two sections. We already noted that turning on fluxes and 'turning on torsion' does not alter the kinetic terms but only induces covariant derivatives and a potential. Since the duality is already 'established' for the kinetic terms in principle, we only need to compare the covariant derivatives and the potential. However, in order to do so we need to first fix the duality map which precisely has to be done for the kinetic terms.

The idea is that for 'every' Calabi-Yau threefold Y there is a whole family of manifolds with SU(3) structure $\hat{Y}_T$ which share the same light spectrum and the same kinetic terms but differ in the torsion T and, therefore, in covariant derivatives and potential. For the case at hand this is expressed by the choice of the parameters q_I^A.

The duality between the heterotic string compactified on $K3 \times T^2$ and the type IIA string compactified on $K3$-fibred Calabi-Yau threefolds has been established mainly in the vector multiplet sector [29 − 33]. It has been shown that for Calabi-Yau threefolds which have the structure of a $K3$ fibred over a $\mathbf{P_1}$ base the vector multiplet couplings can be matched with the heterotic vacuum in the limit of a large $\mathbf{P_1}$. The volume of the $\mathbf{P_1}$ is identified with the heterotic dilaton such that a large $\mathbf{P_1}$ corresponds to weak coupling on the heterotic side. Here we assume that precisely the same situation carries over to manifolds with SU(3) structure and a similar identification can be made. (Obviously it would be very interesting to show this in more detail.) The only caveat is that on the heterotic side the natural variables appearing in a Kaluza-Klein reduction correspond to a parameterization where the prepotential F does not exist. As reviewed in appendix A.1 a symplectic rotation is necessary in order to transform the action into a form that can be compared to the type IIA side. This symplectic rotation is a symmetry for vanishing fluxes but is broken once fluxes are turned on. Therefore it is in general not possible to perform the symplectic rotation in the presence of fluxes. In particular, the rotation exchanges a gauge boson with its magnetic dual which only is a symmetry of the theory if no charged fields are present. The fluxes charge a subset of fields which is another way to see the break down of the symplectic invariance. However, if we choose $m_1^\alpha = 0$ in (2.9) the gauge boson in question drops out of the covariant derivative and the symplectic rotation can be performed.[10]

Let us now turn to the comparison of the hypermultiplet moduli space in the two cases. This is relevant for our analysis as the charged fields obtained by turning on fluxes on the heterotic side, and torsion on the type IIA side, reside in hypermultiplets. Unfortunately, in this sector, the duality is much less understood due to the more complicated structure of the corresponding moduli spaces.[11] For this reason we did not consider the entire hypermultiplet sector, but instead focused on the sub-sector of the $K3$ moduli which span the manifold $\mathcal{M}_H = \mathrm{SO}(4,20)/\mathrm{SO}(4) \times \mathrm{SO}(20)$. This space is what is known to be a dual quaternionic manifold [46] which means that it is in the image of the c-map or in other words it is determined by a special Kähler geometry. For $\mathcal{M}_H$ the underlying special Kähler geometry is the space [46]

$$\mathcal{M}_{SK} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,18)}{\mathrm{SO}(2) \times \mathrm{SO}(18)} \,. \tag{4.1}$$

On the type IIA side we need to make a similar truncation in that out of all hypermultiplets we only keep 20 and they are required to form the same coset space $\mathcal{M}_H$ with a subspace $\mathcal{M}_{SK}$. The $\mathrm{SU}(1,1)/\mathrm{U}(1)$ factor is identified with the type IIA dilaton/axion while the second factor in (4.1) has to be the moduli space of the (truncated) complex structure deformations of the Calabi-Yau threefold and the associated SU(3) structure manifold respectively. So the type IIA coordinates of $\mathcal{M}_{SK}$ are (ϕ, a) and z^a while the 40 RR scalars $(\xi^A, \tilde{\xi}_A)$ are the coordinates which promote $\mathcal{M}_{SK}$ to $\mathcal{M}_H$.

On the heterotic side the coordinates of $\mathcal{M}_H$ are not unique. They are the deformations of the three complex structures, $\vec{J}$, of $K3$, and the internal B-field. The SO(4) rotates these four fields into each other and so, there is also no preferred parameterization of $\mathcal{M}_{SK}$. However, turning on fluxes breaks the SO(4) invariance to SO(3) since they charge the b^α-fields. The charged fields on the type IIA side are the ξ^A which leads us to the identification $b^\alpha \leftrightarrow \xi^A$. The 57 $K3$ moduli which are the expansion coefficients of $\vec{J}$ are then identified with $(z^a, \tilde{\xi}_a)$. However, there is a slight mismatch in that we have 22 b^α but only 20 ξ^A. The reason is that two 'special' heterotic fluxes do not have a geometric IIA dual of the type that we have considered in this paper. This can be seen as follows. The heterotic-type IIA duality constrains the $K3$ of the heterotic compactifications to be a T^2 fibred over a $\mathbf{P_1}$ base or in other words the $K3$ has to be elliptically fibred. The volume of the $\mathbf{P_1}$ base on the heterotic side is identified with the type IIA dilaton/axion. In ref. [34]

[10] Note that here the duals of m_1^α correspond to magnetic charges on the type IIA side. Analyzing such magnetic deformations of manifolds with SU(3) structure goes beyond the scope of this paper and will be dealt with elsewhere [44]. For this reason only we have set the corresponding heterotic fluxes m_1^α to zero. However once these magnetic deformations are identified they are guaranteed to be the duals of m_1^α.

[11] See however ref. [45].

it was shown that fluxes through this $\mathbf{P_1}$ base, which charge the axion with respect to all the gauge fields, correspond to a specific set of RR fluxes on the type IIA side which charge the dual axion again with respect to all vector fields. Apart from this $\mathbf{P_1}$ base, the T^2 fibre is another special cycle among the 22 two-cycles of $K3$. The type IIA dual of this flux we do not understood at present and therefore ignore it in the following.

To summarize, only 20 cycles of $K3$ lead to fluxes which have a geometric IIA dual of the type considered here. For this subset of fluxes only 20 heterotic b-fields become charged and we denote them by b^A. Then the identification above can be refined to

$$b^A\big|_{\rm het} \leftrightarrow \xi^A\big|_{\rm IIA} \ . \tag{4.2}$$

With this identification, the Killing vectors agree if we set $m_I^A = -q_I^A$ and the fluxes through the two special cycles of $K3$ are set to zero.

Finally, let us also compare the two potentials given (2.18) and (3.25). First of all we note that both potentials include the respective dilatons ϕ and both potentials are minimized at weak dilaton coupling $\phi \to -\infty$. This ensures that the supergravity analysis is applicable.

Using the form of the quaternionic metric on the type IIA side, (3.14), we see that we can rewrite the first term in (3.25) as

$$\Big[-\frac{e^{2\phi}}{4}\big(\mathcal{M}\mathrm{Im}\mathcal{M}^{-1}\bar{\mathcal{M}}\big)_{AB} + \frac{e^{4\phi}}{2}\tilde{\xi}_A\tilde{\xi}_B\Big]\,(\mathrm{Im}\mathcal{N}^{-1})^{IJ}\, q_I^A q_J^B = -\frac{1}{2}\, h_{AB}(\mathrm{Im}\mathcal{N}^{-1})^{IJ} q_I^A q_J^B \ , \tag{4.3}$$

where h_{AB} denotes the restriction of the quaternionic metric (3.14) to the charged fields, ξ^A. This shows that the first line in (3.25) precisely matches the first term in the heterotic potential (2.18).

In order to compare the remaining terms we need to use the the special form of the hypermultiplet moduli space $\mathcal{M}_H$ (and $\mathcal{M}_{SK}$) as dictated by the duality. For the geometry (4.1) we can use the formuli of appendix A.1 where the same manifold, (A.6), was discussed for the heterotic vector multiplet sector. Using the form of the matrix $\mathcal{M}$ as given in (A.9) we can explicitely compute the first factor in the second line of (3.25) to be

$$\frac{1}{2}\big(\mathcal{M}\mathrm{Im}\mathcal{M}^{-1}\bar{\mathcal{M}}\big)_{AB} + 4e^{K(z)}\mathcal{G}_A\bar{\mathcal{G}}_B \sim \frac{e^{2\phi}}{4}\,(a^2 + e^{-4\phi})\,\eta_{AB} \ , \tag{4.4}$$

where by η_{AB} we have denoted the invariant symmetric tensor on the second factor of (4.1) and a denotes the type IIA axion. Similarly, using (A.9) again the second term in the second line of (3.25) yields the invariant tensor on the corresponding $\mathrm{SO}(2, n_v-1)$ space

$$\frac{1}{2}\mathrm{Im}\mathcal{N}^{-1\,IJ} + 4X^I\bar{X}^J \sim \frac{1}{Vol(\mathbf{P_1})}\,\eta^{IJ} \ , \tag{4.5}$$

where $Vol(\mathbf{P_1})$ denotes the volume of the $\mathbf{P_1}$ base of the $K3$-fibered Calabi-Yau manifold. In the computation of (4.4) and (4.5) we have not determined the exact numerical factors as this would require establishing a precise map between the fields on both sides. Comparing the second line in (3.25) with the second term in (2.18) using (4.4) and (4.5) we now see that they agree up to undetermined numerical factors.

There is one final thing which can be clarified. According to what we said above we have to identify ρ_{AB} in (2.18) with η_{AB} in (4.4). Recall that ρ_{AB} is the restriction of the $K3$ intersection matrix $\rho_{\alpha\beta}$ where the two special cycles defined by the $\mathbf{P_1}$ base and the T^2 fibre are left out. $\rho_{\alpha\beta}$ has signature $(3,19)$ while η_{AB} (and thus ρ_{AB}) has signature $(2,18)$. The consistency can be checked as follows. The $(3,19)$ signature of $\rho_{\alpha\beta}$ arises from the fact that 3 of the two-forms on $K3$ are selfdual, while the other 19 are anti-selfdual. The forms which are Poincaré dual to the $\mathbf{P_1}$ base and the T^2 fibre can be neither self- nor anti-selfdual since both cycles have zero intersection with itself, while the product of a self-/anti-selfdual form with itself vanishes only if the form itself vanishes.[12] Let us suppose that the Poincaré dual form to the base $\mathbf{P_1}$ has the form $S+A$ where S is selfdual and A is anti-selfdual. For consistency we need that $\int S \wedge S + \int A \wedge A = 0$. Since the elliptic T^2 fibre is the cycle dual to the $\mathbf{P_1}$ base, its Poincaré dual form is the Hodge dual of $S+A$, which, by definition is $S-A$. Therefore, removing the two cycles from our analysis is equivalent to removing the two forms, S and A, from the spectrum and therefore we are left with 2 selfdual and 18 antiselfdual forms. Hence, the restricted intersection matrix ρ_{AB} has signature $(2,18)$ as it should have been for the duality argument to work.

5. Conclusions

In this paper we studied the fate of the non-perturbative duality between heterotic string compactified on $K3 \times T^2$ and type IIA theory compactified on Calabi-Yau threefolds when fluxes on the heterotic side are turned on. We showed that at the level of the effective action one can restore the duality provided on the type IIA side one considers a special class of manifolds with SU(3) structure which were proposed in [8]. Therefore this is one of the few known examples where also a non-perturbative duality seems to hold in the presence of fluxes.

Another aspect of this paper is that embedding manifolds with SU(3) structure satisfying (3.8) into the web string dualities is a further (strong) argument for the existence of these manifolds. This is important to know especially in the fields of moduli stabilization as they induce new terms in the superpotential which can be crucial for moduli stabilization [25, 28, 43, 47].

Finally let us reiterate that there are still many aspects to be better understood in the heterotic-type IIA duality. We already mentioned the largely unknown structure of the hypermultiplet moduli space.[13] Furthermore we only turned on a very special set of heterotic fluxes. In [36] it was shown that turning on fluxes along the heterotic T^2 direction gauges isometries in the scalar manifold of the vector multiplets. So far such gaugings have not been discovered on the type II side and it would be interesting to find this behavior as well. Moreover we have seen that on the type IIA side we have only turned on half of the available geometric fluxes which would mean that additional fluxes should be possible on the heterotic side. We hope to return to some of these issues elsewhere.

[12] Note that this holds for even (anti)self-dual forms on Euclidean spaces, which is precisely our case.

[13] For recent progress in understanding the moduli space of hypermultiplets see for example [48, 49].

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In order to make the paper self-contained we briefly review some of the respective properties of the $N = 2$ low energy effective of the heterotic string compactified on $K3 \times T^2$ (appendix A.1) and type IIA compactified on Calabi-Yau threefolds (appendix B.1). We also show the consistency of $N = 2$ gauged supergravity with the heterotic string in the presence of background fluxes (appendix A.2) and with type IIA compactified on SU(3)-structure manifolds (appendix B.2).

A. $N = 2$ heterotic vacua in four dimensions

This appendix is devoted to the heterotic string compactified on $K3 \times T^2$. Let us first briefly review the properties of the effective action following ref. [36].

A.1 The low energy effective action

The low effective action obtained from the compactifying the heterotic string on $K3 \times T^2$ consists of a $N = 2$ supergravity coupled to n_v (Abelian) vector multiplets each containing a vector, two gaugini and a complex scalar and n_h hypermultiplets each containing two hyperions and four real scalars. The scalars of the vector multiplets consist of the dilaton $e^{-\phi}$, the axion a, the T^2 moduli G_{11}, G_{12}, $\sqrt{G}$ and B_{12}, and the gauge fields in the direction of the torus A_1^a and A_2^a. The correct Kähler coordinates are defined by (see for example [36])

$$s = \frac{a}{2} - \frac{i}{2} e^{-\phi} ,$$

for the dilaton/axion while the others, $t,\ u,\ n^a,\ a = 4, \ldots, n_v$, are given implicitly by the equations[14]

$$\begin{aligned}
A_1^a &= \sqrt{2}\, \frac{n^a - \bar{n}^a}{u - \bar{u}} , \qquad\qquad A_2^a = \sqrt{2}\, \frac{\bar{u} n^a - u \bar{n}^a}{u - \bar{u}} , \\
B_{12} &= \frac{1}{2} \left[(t + \bar{t}) - \frac{(n + \bar{n})^a (n - \bar{n})^a}{u - \bar{u}} \right] , \\
\sqrt{G} &= -\frac{i}{2} \left[(t - \bar{t}) - \frac{(n - \bar{n})^a (n - \bar{n})^a}{u - \bar{u}} \right] , \\
G_{11} &= \frac{2i}{u - \bar{u}} \sqrt{G} , \qquad\qquad G_{12} = i \frac{u + \bar{u}}{u - \bar{u}} \sqrt{G} .
\end{aligned} \tag{A.1}$$

In terms of these coordinates the metric $g_{i\bar{j}}$ is Kähler, with a Kähler potential

$$\begin{aligned}
K &= -\ln i(\bar{s} - s) - \ln \frac{1}{4} \big[(t - \bar{t})(u - \bar{u}) - (n - \bar{n})^a (n - \bar{n})^a \big] \\
&= -\ln i(\bar{s} - s) - \ln X^I \eta_{IJ} \bar{X}^J ,
\end{aligned} \tag{A.2}$$

[14] Note that here a runs only over the Abelian gauge fields (except the KK vectors) which remain massless and not over the whole adjoint representation of the gauge group in ten dimensions.

where the X^I denote the projective coordinates [36, 39]

$$\begin{aligned} X^0 &= \frac{1}{2}\,, \qquad X^1 = \frac{1}{2}(ut - n^a n^a)\,, \quad X^2 = -\frac{1}{2}u\,, \\ X^3 &= \frac{1}{2}t\,, \qquad X^a = \frac{1}{\sqrt{2}}n^a\,. \end{aligned} \tag{A.3}$$

Note that the X^I above satisfy

$$X^I \eta_{IJ} X^J = 0\,, \tag{A.4}$$

where η_{IJ} represents the invariant symmetric tensor of $\mathrm{SO}(2, n_v - 1)$ which in our conventions has the form

$$\eta = \begin{pmatrix} 0 & \mathbf{1}_2 & 0 \\ \mathbf{1}_2 & 0 & 0 \\ 0 & 0 & \mathbf{1} \end{pmatrix}\,. \tag{A.5}$$

The geometry described above, by the Kähler potential (A.2), corresponds to the coset manifold

$$\mathcal{M}_V = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \otimes \frac{\mathrm{SO}(2, n_v - 1)}{\mathrm{SO}(2) \times \mathrm{SO}(n_v - 1)}\,, \tag{A.6}$$

where the first factor is related to the dilaton. As required by $N = 2$ supergravity, this Kähler geometry is in fact a special Kähler geometry. That is, the Kähler potential K can be expressed in terms of the quantities (X^I, F_I) via

$$K = -\ln i\big[\bar{X}^I F_I - X^I \bar{F}_I\big]\,, \tag{A.7}$$

where in a certain symplectic basis F_I can be expressed as $F_I = \frac{\partial F}{\partial X^I}$ with the holomorphic prepotential F being a homogeneous function of degree two in X^I. It is known from ref. [38] that $N = 2$ supersymmetry only requires that F_I exists but not necessarily F itself. For the parameterization given in (A.3) the constraint (A.4) signals that indeed a basis was chosen where no prepotential exists. In refs. [38, 39] it was shown that after the symplectic rotation $X^1 \to -F_1, F_1 \to X^1$ the Kähler potential can be derived from the prepotential

$$F = \frac{X^1(X^2 X^3 - X^a X^a)}{X^0}\,. \tag{A.8}$$

The other couplings of the vector multiplets which appear in the action (2.7) are the gauge couplings $\mathcal{N}$. They are given by

$$\mathcal{N}_{IJ} = -\frac{s + \bar{s}}{2}\eta_{IJ} + \frac{s - \bar{s}}{2}\left(\eta_{IJ} - 2\frac{(X_I \bar{X}_J + \bar{X}_I X_J)}{X^K \eta_{KL} \bar{X}^L}\right)\,, \tag{A.9}$$

where the indices on X are raised and lowered with the metric η defined in (A.5).

Finally, the couplings of the hypermultiplets are encoded in the quaternionic metric h_{uv} of the action (2.7). The scalar fields of the hypermultiplets consist of the $K3$ moduli which span the coset

$$\mathcal{M}_H = \frac{\mathrm{SO}(4, 20)}{\mathrm{SO}(4) \times \mathrm{SO}(20)}\,, \tag{A.10}$$

together with the bundle moduli whose number and couplings can be determined only once a specific solution to (2.2) has been chosen. In the absence of a concrete solution to (2.2) the details of the hypermultiplet moduli space are not known. In the main text we mainly concentrate on the "model independent" part of the metric which is the metric on the moduli space of $K3$.

A.2 Consistency with gauged supergravity

The purpose of this appendix is to show how the potential V given in (2.18) together with the covariant derivatives (2.9) is consistent with the general constraints of gauged $N = 2$ supergravity as for example given in ref. [50]. In fact most of this proof is already contained in ref. [36] where the consistency for $\delta = 0$ was shown. Here we do not want to repeat all the details but merely argue how the proof of [36] carries over to the case $\delta \neq 0$.

The main point is that the covariant derivatives (2.9) do not depend on the value of δ and therefore are unchanged. This in turn leaves the Killing vectors k_I^u defined by $Dq^u = \partial q^u - k_I^u A^I$ unchanged. From (2.9) we infer

$$k_I^u = m_I^\alpha \ . \tag{A.11}$$

In $N = 2$ supergravity the Killing vectors are expressed in terms of a triplet of Killing prepotentials $\mathcal{P}_I^x, x = 1, 2, 3$ defined by $k_I^u K_{uv}^x = -D_v \mathcal{P}_I^x$, where K_{uv}^x is a triplet of almost complex structures which always exists on a quaternionic manifold. Since k_I^u is unchanged for $\delta \neq 0$ also the $\mathcal{P}_I^x$ remain unchanged. The computation of the $\mathcal{P}_I^x$ we do not repeat here but just take them from appendix D of ref. [36]. More precisely let us recall the result[15]

$$\mathcal{P}_I^x \mathcal{P}_J^x = h_{\alpha\beta} m_I^\alpha m_J^\beta + \frac{\rho_{\alpha\beta} m_I^\alpha m_J^\beta}{4v} \ . \tag{A.12}$$

If only isometries in the hypermultiplet sector are gauged the $N = 2$ scalar potential reads [50]

$$V_{N=2} = 4e^K X^I \bar{X}^J h_{uv} k_I^u k_J^v - \left(\frac{1}{2} (\mathrm{Im}\mathcal{N}^{-1})^{IJ} + 4e^K X^I \bar{X}^J\right) \mathcal{P}_I^x \mathcal{P}_J^x \ . \tag{A.13}$$

Inserting (A.11) and (A.12) yields

$$V_{N=2} = -\frac{1}{2} (\mathrm{Im}\mathcal{N}^{-1})^{IJ} h_{\alpha\beta} m_I^\alpha m_J^\beta - \left(\frac{1}{2} (\mathrm{Im}\mathcal{N}^{-1})^{IJ} + 4e^K X^I \bar{X}^J\right) \frac{\rho_{\alpha\beta} m_I^\alpha m_J^\beta}{4v} \ . \tag{A.14}$$

Using the equations (A.2)–(A.9) one finds

$$(\mathrm{Im}\mathcal{N}^{-1})^{IJ} = \frac{2i}{s - \bar{s}} \eta^{IJ} - 4e^K (X^I \bar{X}^J + \bar{X}^I X^J) \ . \tag{A.15}$$

Inserted into (A.14) and using the identity $X^I \eta_{IJ} X^J = 0$ we finally arrive at

$$V_{N=2} = -\frac{1}{2} h_{\alpha\beta} m_I^\alpha m_J^\beta (\mathrm{Im}\mathcal{N}^{-1})^{IJ} + \frac{e^\phi}{4v} \, m_I^\alpha m_J^\beta \rho_{\alpha\beta} \eta^{IJ} \ , \tag{A.16}$$

[15] Note that due to a convention mismatch the second term in this equation came with a minus sign in [36]. However, for $\delta = 0$ this term did not contribute to the potential and therefore its sign was not important in [36].

which indeed coincides with the potential (2.18) derived from the compactification. This establishes the consistency with $N = 2$ gauged supergravity. The second term in (A.16) was missing in [36] due to the constraint $\delta = 0$.

B. $N = 2$ type IIA vacua in four dimensions

B.1 The low energy effective action

We now briefly recall the structure of the $N = 2$ supergravity obtained from Calabi-Yau compactifications of type IIA string theory which was first discussed in [51]. (Our presentation here however follows more closely ref. [42].)

One starts from the ten-dimensional action (3.2) and expands the ten-dimensional fields in terms of the harmonic forms on the Calabi-Yau manifold Y. These are the $h^{(1,1)}$ $(1,1)$-forms ω_i, $i = 1, \ldots, h^{(1,1)}$ and the harmonic three-forms (α_A, β^A), where $A = 0, \ldots, h^{(2,1)}$. There also is a set of dual four-forms $\tilde{\omega}^i$ such that

$$\int_Y \omega_i \wedge \tilde{\omega}^j = \delta_i^j \ . \tag{B.1}$$

Similarly, the harmonic three-forms can be chosen to form a symplectic basis of the third cohomology group H^3 such that

$$\int_Y \alpha_A \wedge \beta^B = \delta_A^B \ , \qquad \int_Y \alpha_A \wedge \alpha_B = \int_Y \beta^A \wedge \beta^B = 0 \ . \tag{B.2}$$

The ten-dimensional massless fields $\hat{B}_2, \hat{C}_1, \hat{C}_3$ are then Kaluza-Klein expanded according to

$$\begin{aligned} \hat{B}_2 &= B_2 + b^i \omega_i \ , \\ \hat{C}_1 &= A^0 \ , \\ \hat{C}_3 &= C_3 + A^i \wedge \omega_i + \xi^A \alpha_A - \tilde{\xi}_A \beta^A \ , \end{aligned} \tag{B.3}$$

where B_2 is a two form, (A^0, A^i) are one-forms and $b^i, \xi^A, \tilde{\xi}_A$ are scalars. Furthermore the Calabi-Yau metric has two sets of independent deformations, the deformations v^i, of the Kähler form J, and the deformations z^a, of the complex structure, which can equivalently be viewed as the deformations of the holomorphic three-form Ω

$$J = v^i \omega_i \ , \qquad \Omega = Z^A \alpha_A - \mathcal{G}_A \beta^A \ . \tag{B.4}$$

$\mathcal{G}_A = \frac{\partial \mathcal{G}}{\partial Z^A}$ is the derivative of the holomorphic prepotential $\mathcal{G}$ and the complex structure moduli are given by $z^a = Z^a / Z^0$. Altogether, these fields combine into $h^{(1,1)}$ vector multiplets, consisting of the bosonic components $(A^i, x^i = b^i + iv^i)$ and $h^{(2,1)}$ hypermultiplets consisting of the scalars $(z^a, \xi^a, \tilde{\xi}_a)$. In addition there is a tensor multiplet with components $(B_2, \phi, \xi^0, \tilde{\xi}_0)$ and finally A^0 is the graviphoton which sits in the $N = 2$ gravitational multiplet.

If the tensor multiplet is dualized to an additional hypermultiplet, the effective action is again of the standard $N = 2$ form as given in (2.7). The metric g_{ij} of the scalars in the vector multiplets is defined as

$$g_{ij} = \frac{1}{4\mathcal{K}} \int_Y \omega_i \wedge *\omega_j \ , \tag{B.5}$$

where $\mathcal{K}$ is the volume of Y. As required by $N = 2$ supergravity this metric is special Kähler and can be derived from the prepotential

$$F = -\frac{1}{6}\mathcal{K}_{ijk}x^i x^j x^k \ , \tag{B.6}$$

where $\mathcal{K}_{ijk}$ denotes the triple intersection numbers of the Calabi-Yau manifold. Furthermore, the imaginary part of the gauge coupling matrix $\mathcal{N}$ in (2.7) is given by[16]

$$\mathrm{Im}\mathcal{N} = -\mathcal{K}\begin{pmatrix} 1 + 4g_{ij}b^i b^j & 4g_{ij}b^j \\ 4g_{ij}b^i & 4g_{ij} \end{pmatrix} . \tag{B.7}$$

On the type IIA side the quaternionic metric h_{uv} on the space of hypermultiplet scalars can be given explicitly. It is determined in terms of the special Kähler geometry which describes the complex structure deformations z^a on a Calabi-Yau manifold. One first defines the matrix $\mathcal{M}_{AB}$ by the following integrals[17]

$$\begin{aligned}
\int_Y \alpha_A \wedge *\alpha_B &= -\,\mathrm{Im}\mathcal{M}_{AB} - \mathrm{Re}\mathcal{M}_{AC}(\mathrm{Im}\mathcal{M})^{-1\,CD}\mathrm{Re}\mathcal{M}_{DB}\ , \\
\int_Y \alpha_A \wedge *\beta^B &= -\,\mathrm{Re}\mathcal{M}_{AC}(\mathrm{Im}\mathcal{M})^{-1\,CB}\ , \\
\int_Y \beta^A \wedge *\beta^B &= -\,(\mathrm{Im}\mathcal{M})^{-1\,AB}\ .
\end{aligned} \tag{B.8}$$

The quaternionic metric is then expressed in terms of $\mathcal{M}_{AB}$ by [52]

$$\begin{aligned}
h_{uv}Dq^u \wedge *Dq^v = &\ \frac{1}{4}(d\phi)^2 + g_{a\bar{b}}dz^a \wedge *d\bar{z}^b + \frac{e^{4\phi}}{4}\left[Da - (\tilde{\xi}_A D\xi^A - \xi^A D\tilde{\xi}_A)\right]^2 \\
&- \frac{e^{2\phi}}{2}\left(\mathrm{Im}\mathcal{M}^{-1}\right)^{AB}\left[D\tilde{\xi}_A - \mathcal{M}_{AC}D\xi^C\right] \wedge *\left[D\tilde{\xi}_B - \bar{\mathcal{M}}_{BD}D\xi^D\right] .
\end{aligned} \tag{B.9}$$

where, $g_{a\bar{b}}$ is the metric on the complex structure moduli space and a denotes the (pseudo)-scalar which is dual to the two-form B_2 in four dimensions.

B.2 Consistency with gauged supergravity

The purpose of this appendix is to show the consistency of the potential (3.25) with the general form of the $N = 2$ potential (A.13). Note that this can be written as

$$V_{N=2} = \left[\frac{1}{2}(\mathrm{Im}\mathcal{N}^{-1})^{IJ} + 4e^{K(X)}X^I\bar{X}^J\right](h_{uv}k_I^u k_J^v - \mathcal{P}_I^x\mathcal{P}_J^x) - \frac{1}{2}(\mathrm{Im}\mathcal{N}^{-1})^{IJ}k_I^u k_J^v h_{uv}\ . \tag{B.10}$$

[16]The real part of this matrix plays no role for the analysis here but can be found, for example, in ref. [42].

[17]Note that due to the Hodge $*$ these integrals depend on the choice of complex structure or in other words on z^a.

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As in the heterotic case we first need to determine the Killing prepotentials $\mathcal{P}_I^x$. For generic manifolds with SU(3) structure they were computed in ref. [8]. For the case at hand they read

$$\mathcal{P}_I^1 + i\mathcal{P}_I^2 = 2e^{\frac{1}{2}K(z)+\phi} q_I^A \mathcal{G}_A\,, \qquad \mathcal{P}_I^3 = e^{2\phi} q_I^A \tilde{\xi}_A\,. \tag{B.11}$$

This enables us to also compute

$$\mathcal{P}_I^x \mathcal{P}_J^x = 4e^{K(z)} e^{2\phi} \mathcal{G}_A \bar{\mathcal{G}}_B q_I^A q_J^B + e^{4\phi} \tilde{\xi}_A \tilde{\xi}_B q_I^A q_J^B\,. \tag{B.12}$$

Inserted into (B.10) and also using $k_I^u = -q_I^A$ as can be seen from (3.17) and h_{uv} given in (3.14) it is now not hard to see that the above potential precisely coincides with the one obtained in (3.25). This establishes that the result on the type IIA side does indeed describe a $N = 2$ gauged supergravity.

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